Mesh Stiffness of Cylindrical Gear Including Tooth Surface Friction Based on FEA Method

Liu Geng, Nan Mimi, Liu Lan, Zhao Ying, Wu Liyan,
Shaanxi Engineering Laboratory for Transmissions and Controls, Northwestern Polytechnical University, Xi’an, China

Abstract
The tooth friction and oil film stiffness should be considered into the time-varying meshing stiffness to obtained more precise mesh stiffness. The paper presents a method to include the effect of friction and lubrication into mesh stiffness of cylindrical involute gears using a combination of finite element method (FEM) and local contact analysis of elastic bodies. The normal compliances matrix and normal-tangential compliances matrix are obtained with a finite element model by using substructure method, and the tooth contact deformations are derived through an elastic line-contact analysis. The oil film thickness and stiffness are calculated by theoretical formulas. The time-varying stiffness of the gear pair can be obtained by solving the nonlinear deformation compatibility equations. The mesh stiffness of spur and helical gears with or without friction are compared in this paper. The effects of loads and friction coefficient on mesh stiffness of helical gears are also investigated.

Key words: Mesh Stiffness, FEA, Oil Film Stiffness, Cylindrical gear

1 Introduction
The method proposed in this paper is based on the work done by Chang [1-2] who has determined the load distribution in the meshing plane and the mesh stiffness without friction and oil film stiffness. The recent researches [3-5] focus on the time-varying mesh stiffness including the effect of tooth surface properties (friction or lubrication), however, the methods they utilized is based on mechanics of material regarding the gear tooth as a cantilever beam.
which may cause some deviations and when the method is applied to helical gears more deviations would be induced due to the independence between every thin gear slice. However FEM provides full consideration for the influence between the gear body and gear teeth.

In the present study, a method to determine mesh stiffness including tooth friction and oil film stiffness of cylindrical gears is proposed based on FEM and elastic contact theory. On one hand FEM has an obvious advantage in predicting global deflection as well as the influence between the normal direction and tangential direction of the tooth surface, on the other hand the contact theory can predict local contact deformation accurately, the proposed method combines the advantages of each to increase the computation efficiency and provide sufficient precision in predicting mesh stiffness.

2 The Time-varying Mesh Stiffness Model Including Tooth Friction and Oil Film Stiffness

2.1 Meshing stiffness model

Considering the squeezing deformation of oil between gear teeth, the meshing stiffness include two parts namely gear teeth stiffness and oil film stiffness, and the relationship of them is illustrated in Fig. 1. The meshing stiffness model proposed here is based on the time-varying meshing stiffness model without considering tooth friction and oil film proposed by Chang et al [1-2]. The combination of FE method and contact theory is adopted to calculate the meshing stiffness for the sake of computation efficiency and provide sufficient precision. And the finite element model is shown in Fig. 2. And the bending deformation is separated from the total deformation in the FE model, see Fig. 3, and the bending compliances of contact points are derived using two-dimensional interpolation from the compliances of nodes on tooth surface, see Fig. 4.
Then the contact deformation of each contact point is obtained by using an analytical equation see Eq. (1) for a line-contact elastic contact deformation [6]:

$$\delta_c = \frac{n \eta p}{m_l} \ln \frac{6.59^{1}(R_1 + R_2)}{\eta p R_1 R_2}$$

(1)

where $\delta_c$ is the contact deformation of point $i$, $l_i$ is the length of subsection contact line, $R_1$ and $R_2$ are the normal radii of curvature for pinion and wheel, $\eta = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$, and $E_1$ and $E_2$ are the Young's modulus, $\nu_1$ and $\nu_2$ are the Poisson's ratio.

Since the deformation at each contact point is separated into global bending term, local contact term, friction deformation term and oil deformation term. Without regarding to tooth shape deviation and to ensure the continuity of engagement, the deformations compatibility relationship of all contact points at the same meshing time is expressed in Eq. (2), that is:

$$\begin{bmatrix}
    f_{b11} & f_{b12} & \cdots & f_{b1n} & p_1 \\
    f_{b21} & f_{b22} & \cdots & f_{b2n} & p_2 \\
    \vdots & \vdots & \cdots & \vdots & \vdots \\
    f_{bn1} & f_{bn2} & \cdots & f_{bn1} & p_n
\end{bmatrix} + \begin{bmatrix}
    f_{c11} & f_{c12} & \cdots & f_{c1n} & \mu_{c1} \\
    f_{c21} & f_{c22} & \cdots & f_{c2n} & \mu_{c2} \\
    \vdots & \vdots & \cdots & \vdots & \vdots \\
    f_{cn1} & f_{cn2} & \cdots & f_{cnn} & \mu_{cn}
\end{bmatrix} = \begin{bmatrix}
    p_1 + [\delta_{c1}] \\
    p_2 + [\delta_{c2}] \\
    \vdots \\
    p_n + [\delta_{cn}]
\end{bmatrix} + \begin{bmatrix}
    \delta_{c1} \\
    \delta_{c2} \\
    \vdots \\
    \delta_{cn}
\end{bmatrix}$$

(2)
where \( f_{bij} \) is the bending compliance, which means the bending deformation at point \( i \) as a result of a unitary normal load at point \( j \), \( f_{tij} \) is the bending compliance, which means the bending deformation at point \( i \) as a result of a unitary tangential load at point \( j \), \( \mu_{nn} \) is friction coefficient at point \( n \), the \( \delta_{ci} \) is the contact deformation at point \( j \), \( \delta_{oi} \) is the normal contact deformation of the oil film at point \( j \), \( p_{j} \) is the normal load at point \( j \), \( n \) is the total number of contact points, \( C \) is the total normal deformation at each point, which is the transmission error regardless of shape deviation.

The force equilibrium relationship can be expressed as:

\[
\sum_{j=1}^{n} p_{j} = P
\]  

(3)

where \( P \) is the total normal external mesh force. The distributed load \( p_{i} \) and total deformation \( C \) can be obtained by solving the nonlinear Eq. (2) and Eq. (3) using numerical iterative method. Then the mesh stiffness \( C_{\gamma} \) is determined by

\[
C_{\gamma} = \frac{P}{(C \cdot B)}
\]  

(4)

The time varying mesh stiffness in a mesh period will be obtained by repeating the process in the other mesh positions.

2.2 Tooth surface friction coefficient and oil film stiffness

There are two terms in Eq. (2) to include the influence of the oil lubrication and the friction existed between contact surfaces. So a convenient EHL model proposed by XU et al [7] is applied to calculate the time-varying friction coefficient approximately; the formula is expressed as follows:

\[
\mu_{nn} = \mathrm{sgn}(SR)e^{P_{h}h^{b_{1}}SR^{b_{2}}}u_{h}^{b_{3}}v_{0}^{b_{4}}R^{b_{5}}
\]  

(5)

\[
t = b_{1} + b_{2}[SR/P_{h} \log_{10}(v_{0})] + b_{3}e^{-f_{p}P_{h}\log_{10}(v_{0})} + b_{4}e^{t}
\]

where \( SR \) is the slide-to-roll ratio, and \( P_{h} \) is the line-contact maximum Hertzian pressure at contact points, \( v_{0} \) is the absolute viscosity, \( u_{h} \) is denoted the entrainment velocity and \( S \) is the RMS composite surface roughness, and \( b_{i} \) (i=1, 2,…, 9) are constant coefficients and \( b_{i}=-8.92, \)
1.03, 1.04, -0.35, 2.81, -0.10, 0.75, -0.39, and 0.62 for i=1~9, respectively. Since this formula is proposed for spur gears, here we regard the every finite element as a spur gear.

Oil film stiffness is determined by the lubrication theory, to simplify the computation the minimum oil film thickness is calculated by the formula proposed by Dowson and Higginson, see Eq. (6):

\[
h_{\text{min}} = 2.65 G^{0.54} U^{0.7} W^{-0.13} R
\]

where \( W = w/(E^* R) \), \( U = v_0 u_0/(E^* R) \), \( G = \alpha E^* \), \( W \), \( U \), \( G \) are the load parameter and material parameter, respectively, \( R \) is the equivalent contact radius at the contact point, \( 1/E^* = 0.5[(1-v_1^2)/E_1+(1-v_2^2)/E_2] \). And the center oil film thickness \( h_c = 4/3 h_{\text{min}} \) pointed out by Dowson and Higginson. Since the contact radius represents the center of line contact and the Eq. (1) is used to calculate the center contact deformation, the center oil film thickness is adopted to calculate the Oil film stiffness which is based on the relationship between normal load and film thickness defined by Huang et al [8] as given as Eq. (7):

\[
\delta_{oi} = 1/K_{oi} = 1/(0.84 LE^* R(2.65 G^{0.54} U^{0.7} R)^{7.66} h_c^{-8.69})
\]

3 Results and discussion

The method presented in this study is applied to cylindrical gears, and two types of gear parameters are taken as examples and the gear parameters are listed in Table 1 and Table 2.

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<tr>
<th>z₁</th>
<th>z₂</th>
<th>m₀/mm</th>
<th>α₀/(°)</th>
<th>β/(°)</th>
<th>B/mm</th>
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<td>37</td>
<td>62</td>
<td>2.5</td>
<td>20</td>
<td>0</td>
<td>34</td>
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<table>
<thead>
<tr>
<th>z₁</th>
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<th>β/(°)</th>
<th>B/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>62</td>
<td>2.5</td>
<td>20</td>
<td>5,15,25</td>
<td>34</td>
</tr>
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</table>
3.1 Results for spur gears

The deformations of pinion and gear with different friction coefficient models and without friction are shown in Fig. 5 and it can be concluded that friction has different effects on the deformation of the pinion and gear and the changes of friction coefficients at pitch point will influence the mesh stiffness at pitch point.

![Fig. 5: Deformations of pinion and gear with different friction coefficient models](image)

The mesh stiffnesses for spur gears with different models of friction coefficient and without friction under different loads are shown in Fig. 6. As can be seen from Fig. 6 a), the wave curve of mesh stiffness with Xu friction model is smoother than that with friction coefficient 0.1 for the former one changes more smoothly at pitch point, and the oil makes the big difference to the mesh stiffness. Fig 6 b) shows that higher load causes higher mesh stiffness, the oil also decreases the mesh stiffness.

![Fig. 6: Mesh stiffness with a) different surface models, b) different loads](image)

3.2 Results for helical gears

The mesh stiffnesses for helical gears with different models of friction coefficient and without friction under different loads are shown in Fig. 7. As can be seen from Fig. 7, the friction increases the mesh stiffness and oil film decreases the mesh stiffness, and the effect of oil
film is more obvious, and the mesh stiffnesses for helical gears with different helical angles 
are shown in Fig. 8, and it can be seen that tooth friction and oil film can decrease the value of 
mesh stiffness.

![Fig. 7: Mesh stiffness for helical gear (helical angle 10°) with a) different friction model, b) different loads](image)

![Fig. 8: Mesh stiffness with different helix angles](image)

4 Conclusion

This study presents a new method including the effect of tooth friction and oil film in mesh 
stiffness of cylindrical gears. The application of substructure in FE model and the decoupling 
of load and tooth friction make this method suitable for cylindrical gears. It is investigated that 
the tooth friction can increase mesh stiffness and the bigger the coefficient is, the more ob-
vvious the influence can be seen. And it is also found that the oil film can decrease the mesh 
stiffness for the relationship between oil film stiffness and stiffness of gear and pinion is series 
connection, so smaller oil film stiffness can have bigger effect on mesh stiffness.
References


