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M.Sc. Simeon Hubrich,
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# The hierarchical finite cell method for nonlinear problems: Moment fitting quadratures, basis function removel, and remeshing

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In this thesis, several approaches are discussed in order to further enhance the performance of the finite cell method (FCM). Thereby, novel moment fitting quadrature schemes are introduced that allow to reduce the effort of the numerical integration process significantly. Further, a basis function removal scheme is proposed to improve the conditioning behavior of the resulting equation system. Finally, an innovative remeshing strategy is presented that overcomes the problem of severely distorted elements for simulations with large deformations.

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*To my love Anna*
*and our beautiful daughter Clari,*
*I love you!*

# Contents

# Abstract

Over the last decade, nonstandard discretization methods based on the fictitious domain approach have gained increased interest. In these methods, the physical domain is embedded into a fictitious one – resulting in an extended domain of a simple shape. Consequently, structured meshes or Cartesian grids can be employed for the spatial discretization, thus simplifying the mesh generation process significantly. Due to this reason, such methods are a powerful tool for the numerical analysis of complex structures such as foam-like materials. A well-known example for these methods is the *finite cell method* (FCM), which combines the fictitious domain approach with high order finite elements. In the FCM, these elements are denoted as finite cells – thus giving the method its name – in order to distinguish them from boundary-conforming finite elements. However, the simplification in the mesh generation is accompanied by several numerical difficulties, induced by cut finite cells, reducing the efficiency and robustness of the FCM. In this thesis, we focus on the following issues in order to further improve the FCM.

The first topic is related to the **numerical integration of finite cells**. In general adaptive Gaussian quadrature schemes are used – commonly resulting in a large number of integration points, which renders the numerical integration computationally expensive. To overcome this problem, we propose novel quadrature methods based on **moment fitting**. Thereby, a promising approach is introduced that circumvents the necessity of having to solve an equation system. We show that this moment fitting method results in efficient and accurate quadrature rules for linear problems of the FCM, reducing the effort during the numerical integration process significantly. Moreover, in order to improve the performance for nonlinear applications, an adaptive moment fitting approach is presented.

The second topic addresses the **ill-conditioning of the global system**. To improve the conditioning behavior, we propose a new **basis function removal** approach applied to the hierarchic shape functions of the FCM. In this approach, shape functions with a small contribution to the diagonal entries of the global system matrix are removed from the ansatz. To this end, a global criterion based on the discrete gradient operator is introduced to estimate the contribution. Moreover, by maintaining the nodal modes of the hierarchic shape functions, the modified basis preserves the representation of the rigid body modes. Several examples show that the basis functions removal improves the conditioning behavior and, thus, the performance of the FCM significantly.

The last topic is related to the issue of **severely distorted finite cells for applications in finite strain**. To overcome this problem, we introduce a novel **remeshing strategy** that is based on a multiplicative decomposition of the deformation gradient. The essential idea of this strategy is to create a new mesh whenever the analysis fails due to severe distortions of the computational mesh – and then to continue the simulation. Further, a local radial basis function interpolation scheme for the implementation of the data transfer is presented. Considering problems of different complexity, we show that the remeshing strategy allows to improve the robustness behavior of the FCM considerably, especially in combination with the presented basis function removal.