

Reihe 1

Konstruktions-  
technik/  
Maschinen-  
elemente

Nr. 438

Dipl.-Wirtsch.-Ing. (Univ.) Benjamin Schleich,  
Erlangen

## Skin Model Shapes: A new Paradigm for the Tolerance Analysis and the Geometrical Variations Modelling in Mechanical Engineering

Lehrstuhl für

**Konstruktionstechnik**

Friedrich-Alexander-Universität Erlangen-Nürnberg  
Prof. Dr.-Ing. Sandro Wartzack

<https://doi.org/10.51202/bf03186438010-1>

Generiert durch IP '3.144.251.107' am 20.05.2024, 21:22:30

Das Erstellen und Weitergeben von Kopien dieses PDFs ist nicht zulässig.





**Skin Model Shapes:**  
**A new Paradigm for the Tolerance Analysis**  
**and the Geometrical Variations Modelling**  
**in Mechanical Engineering**

Skin Model Shapes: Ein neues Paradigma für die Toleranzanalyse  
und die Modellierung geometrischer Abweichungen im Maschinenbau

Der Technischen Fakultät  
der Friedrich-Alexander-Universität  
Erlangen-Nürnberg  
zur Erlangung des Doktorgrades

**DOKTOR-INGENIEUR**

vorgelegt von  
Benjamin Schleich  
aus Fürth

Als Dissertation genehmigt von  
der Technischen Fakultät der  
Friedrich-Alexander-Universität Erlangen-Nürnberg

Tag der mündlichen Prüfung: 20. März 2017  
Vorsitzender des Promotionsorgans: Prof. Dr.-Ing. Reinhard Lerch

Gutachter: Prof. Dr.-Ing. Sandro Wartzack  
Prof. Luc Mathieu

# Fortschritt-Berichte VDI

Reihe 1

Konstruktionstechnik/  
Maschinenelemente

Dipl.-Wirtsch.-Ing. (Univ.)  
Benjamin Schleich, Erlangen

Nr. 438

**Skin Model Shapes:  
A new Paradigm for  
the Tolerance Analysis  
and the Geometrical  
Variations Modelling in  
Mechanical Engineering**

Lehrstuhl für

**Konstruktionstechnik**

Friedrich-Alexander-Universität Erlangen-Nürnberg  
Prof. Dr.-Ing. Sandro Wartzack



Schleich, Benjamin

### **Skin Model Shapes:**

### **A new Paradigm for the Tolerance Analysis and the Geometrical Variations Modelling in Mechanical Engineering**

Fortschr.-Ber. VDI Reihe 1 Nr. 438. Düsseldorf: VDI Verlag 2017.

256 Seiten, 173 Bilder, 19 Tabellen.

ISBN 978-3-18-343801-3, ISSN 0178-949X,

€ 90,00/VDI-Mitgliederpreis € 81,00.

**Für die Dokumentation:** Simulation – Toleranzanalyse – Toleranzmanagement – Produktentwicklung – Konstruktion – Robust Design – Unsicherheit – Dimensional Management – Geometrische Produktspezifikation – Skin Model Shapes

Die vorliegende Arbeit wendet sich an Ingenieure und Wissenschaftler aus dem Bereich der virtuellen Produktentwicklung und angrenzender Disziplinen mit Schwerpunkt auf dem Toleranzmanagement. Sie stellt ein umfassendes Rahmenwerk für die Toleranzsimulation unter Berücksichtigung von Formabweichungen in Übereinstimmung mit internationalen Normen zur Geometrischen Produktspezifikation vor. Hierzu werden Algorithmen für die Erzeugung abweichungsbehafteter Punktwolken und Oberflächennetze (Skin Model Shapes), für deren Skalierung sowie deren Montage-simulation erarbeitet. Darüber hinaus behandelt die Arbeit die Toleranzanalyse bewegter Mechanismen und stellt einen Software-Prototypen für die Toleranzsimulation mittels Oberflächennetzen vor. Auf Basis der kritischen Gegenüberstellung von Ergebnissen des erarbeiteten Verfahrens mit bestehenden Toleranzanalysemethoden für typische Problemfälle wird gezeigt, dass durch die Berücksichtigung von Formabweichungen bei der Toleranzanalyse die Einflüsse von Bauteilabweichungen auf die Funktion und Qualität mechanischer Baugruppen und bewegter Systeme deutlich realitätsnäher bestimmt und dadurch Toleranzentscheidungen optimiert werden können.

### **Bibliographische Information der Deutschen Bibliothek**

Die Deutsche Bibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliographie; detaillierte bibliographische Daten sind im Internet unter <http://dnb.ddb.de> abrufbar.

### **Bibliographic information published by the Deutsche Bibliothek**

(German National Library)

The Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliographie (German National Bibliography); detailed bibliographic data is available via Internet at <http://dnb.ddb.de>.

© VDI Verlag GmbH · Düsseldorf 2017

Alle Rechte, auch das des auszugsweisen Nachdruckes, der auszugsweisen oder vollständigen Wiedergabe (Fotokopie, Mikrokopie), der Speicherung in Datenverarbeitungsanlagen, im Internet und das der Übersetzung, vorbehalten.

Als Manuskript gedruckt. Printed in Germany.

ISSN 0178-949X

ISBN 978-3-18-343801-3

## Preface

The current work was developed during my time as an academic counsellor at the Institute of Engineering Design KTMfk of the FAU Erlangen-Nürnberg. During that time, I received great support from and I was influenced by many persons, to whom I am sincerely grateful.

Particularly, I thank Prof. Dr.-Ing. Sandro Wartzack for offering me the opportunity to conduct my work under his guidance at the Institute of Engineering Design, for his professional supervision, and his constant support in scientific as well as personal affairs. The last five years have been not only busy, productive, and educational, but they have also been a pleasant expedition covering successful projects, minor detours, and much fun. I deeply thank Sandro for shaping this time and for providing me with responsibilities, scope for development, and unconditional trust during this period.

Moreover, I thank Prof. Luc Mathieu as a highly acknowledged expert and researcher in tolerancing and production engineering for sharing his professional expertise and personal advice, for contributing numerous valuable suggestions during the last years, and for agreeing to act as the co-examiner.

Additionally, I thank Prof. Dr. sc. ETH Alexander Hasse for his interest in my work and for acting as the chairperson of the examination committee. I also thank Prof. Dr. Michael Stingl for his interest in my work and for acting as a further member of the examination committee.

Beside this, my special thank goes to all past and present colleagues of the KTMfk, particularly to the tolerancing and robust design group, for the countless professional discussions, the constructive feedback, as well as for the optimal technical and collegial working environment. Only the working in such a strong and dedicated team allowed us to master many challenging tasks and projects. Moreover, I want to express my deep appreciation to those students, who have contributed to this work in any manner.

Furthermore, I want to express my deepest gratitude to Dr. Nabil Anwer for taking the time for numerous discussions and web-meetings during the last years, for sharing his professional experience, and for the fruitful and friendly cooperation.

I also thank all colleagues from national and international research laboratories and the computer-aided tolerancing community, who encouraged me in my work. Particularly, I thank Dr. Alex Ballu as one of the conceptual fathers of the Skin Model, Prof. Jean-Yves Dantan, and Prof. Rikard Söderberg for their favourable support and encouragement as well as many discussions and conversations during conferences and meetings. I also like to offer my thanks to all colleagues and partners from industry, who provided me with valuable comments and feedback as well as encouragement during the last years.

Last but most important, I owe my sincere and deepest gratitude to my beloved Juli, my parents, my brother, my sister-in-law, and the rest of my family for their patience and for providing me with unconditional love, support, motivation, and welcome diversion. All the efforts and hardships would not have been worthwhile without you.

March 2017

Benjamin Schleich

– *Meinen Eltern* –



# Contents

<b>Abbreviations and Symbols</b>	<b>VIII</b>
<b>Zusammenfassung</b>	<b>XII</b>
<b>Abstract</b>	<b>XIII</b>
<b>1 Introduction</b>	<b>1</b>
1.1 The Context: Geometrical Variations Management . . . . .	1
1.2 The Essence: The Concept of Skin Model Shapes as a new Paradigm for Modelling Geometrical Variations . . . . .	2
1.3 The Aim: A Framework for the Tolerance Analysis based on Skin Model Shapes	3
1.4 The Approach: Scope and Outline of the Work . . . . .	4
<b>2 Context and State of the Art</b>	<b>5</b>
2.1 Geometrical Variations Management throughout the Product Life-Cycle . . . . .	5
2.2 Standards for the Geometrical Product Specification as the predominant Means of Communication in Geometrical Variations Management . . . . .	10
2.2.1 A Brief History of the Standards for the Geometrical Product Specification . . . . .	12
2.2.2 Fundamentals and Basic Concepts of modern GPS Standards . . . . .	17
2.2.3 Recent Trends regarding the GPS Standards . . . . .	26
2.2.4 Further Approaches for the Specification of Geometrical Requirements	27
2.3 Tolerancing as an integral Part of Geometrical Variations Management . . . . .	28
2.3.1 Disambiguation and Definition . . . . .	29
2.3.2 Tolerancing Activities throughout the Product Life-Cycle . . . . .	31
2.3.3 Computer-Aided Tolerance Analysis . . . . .	45
<b>3 Identification of Need for Research</b>	<b>63</b>
3.1 Discussion of the State of the Art . . . . .	63
3.2 Research Gap and Scientific Challenge . . . . .	64
3.3 Further Outline of the Work . . . . .	66
<b>4 The Concept of Skin Model Shapes as a new Paradigm for the Modelling of Geometrical Variations</b>	<b>67</b>
4.1 Motivation and Model Conceptualisation . . . . .	67
4.2 Representation and Visualisation of Skin Model Shapes . . . . .	69
4.3 Generation of Skin Model Shapes . . . . .	71
4.3.1 Related Work . . . . .	71
4.3.2 Overall Framework . . . . .	73
4.3.3 Skin Model Shape Generation in the Prediction Stage . . . . .	74
4.3.4 Skin Model Shape Generation in the Observation Stage . . . . .	78

4.3.5	Comparison of the Approaches for the Generation of Skin Model Shapes	81
4.4	Applications and Perspectives for Skin Model Shapes in Mechanical Engineering	83
<b>5</b>	<b>A comprehensive Framework for the Computer-Aided Tolerance Analysis based on Skin Model Shapes</b>	<b>85</b>
5.1	Framework for the Tolerance Analysis based on Skin Model Shapes	85
5.2	Scaling of Skin Model Shapes	86
5.2.1	Related Work	87
5.2.2	Scaling Sequence based on the Relationship between different Tolerances	88
5.2.3	Dimensional Tolerances	90
5.2.4	Form Tolerances	92
5.2.5	Orientation Tolerances	96
5.2.6	Location Tolerances	102
5.2.7	Profile and Runout Tolerances	106
5.2.8	Alternative Tolerance Specifications	107
5.2.9	Results for the Example Case Study	109
5.3	Relative Positioning and Assembly Simulation of Skin Model Shapes	110
5.3.1	Related Work	111
5.3.2	Relative Positioning of Skin Model Shapes	112
5.3.3	Assembly Simulation for Skin Model Shapes	119
5.3.4	Application to the Example Case Study	129
5.4	Gap Hull Estimation for Skin Model Shapes	130
5.4.1	Related Work	130
5.4.2	Problem Description and General Approach	131
5.4.3	Exemplary Application	134
5.5	Contact and Mobility Simulation for Gears and Rotating Mechanism	137
5.5.1	Related Work	137
5.5.2	Approaches for the Contact and Mobility Simulation of Skin Model Shapes	138
5.6	Measurement of Key Characteristics and Result Visualization	152
5.6.1	Approaches for the Measurement of Key Characteristics	152
5.6.2	Result Visualization Methods	152
5.6.3	Application to the Example Case Study from Section 5.1	153
<b>6</b>	<b>Prototype Implementation of a Tolerance Analysis Tool based on Skin Model Shapes</b>	<b>157</b>
6.1	General Architecture and Workflow	157
6.2	Part Import Module	158
6.3	Tolerance Specification and Assembly Definition Module	159
6.4	Tolerance Simulation Module	160
6.5	Result Visualization and Export Module	160
6.6	Additional Functionalities	160

---

<b>7 Application and Benchmark of the Tolerance Analysis Approach based on Skin Model Shapes</b>	<b>163</b>
7.1 Tolerance Stack-Ups . . . . .	163
7.1.1 Tolerance Stack-Up of two Cubes . . . . .	163
7.1.2 Tolerance Stack-Up of two Plates and a Cube . . . . .	167
7.1.3 Tolerance Stack-Up of four Parts . . . . .	170
7.1.4 Tolerance Stack-Up considering different Positioning Schemes . . . . .	174
7.1.5 Tolerance Stack-Up of four Parts considering 3-2-1 Positioning Schemes	175
7.2 Product Assemblability Evaluation . . . . .	176
7.2.1 Pin-Hole Assembly . . . . .	177
7.2.2 Two-Pin-Two-Hole Assembly . . . . .	179
7.3 Further Applications and Additional Functionalities . . . . .	182
7.3.1 Five-Piece Assembly . . . . .	182
7.3.2 Two Discs in a Box . . . . .	187
7.3.3 Irregular Tolerance Stack . . . . .	188
7.4 Discussion of the obtained Results . . . . .	190
<b>8 Conclusion and Perspectives for future Research</b>	<b>192</b>
<b>Appendix</b>	<b>194</b>
<b>Bibliography</b>	<b>199</b>
<b>Publications by the Author</b>	<b>239</b>

## Abbreviations and Symbols

### *Abbreviations*

ANOVA	Analysis of Variance
ASME	American Society of Mechanical Engineers
BC	Before Christ
BRep	Boundary Representation
CAD	Computer-Aided Design
CAM	Computer-Aided Manufacturing
CAPP	Computer-Aided Process Planning
CAT	Computer-Aided Tolerancing
CLTE	Closed-Loop Tolerance Engineering
CMM	Coordinate Measuring Machine
CSG	Constructive Solid Geometry
CZ	Common Zone
D	Dimension
DIN	Deutsches Institut für Normung (German Institute for Standardization)
DLM	Direct Linearisation Method
DoF	Degree of Freedom
E. g.	Exempli Gratia
(E)FAST	(Extended) Fourier Amplitude Sensitivity Test
FEA	Finite Element Analysis
GD&T	Geometric Dimensioning and Tolerancing
GPS	Geometrical Product Specification
GUI	Graphical User Interface
I. a.	Inter alia
I. e.	Id est
I. i. d.	Independently and Identically Distributed
ICDF	Inverse Cumulative Distribution Function
ICP	Iterative Closest Point
IGES	Initial Graphics Exchange Specification
ISO	International Organization for Standardization
ITP	Integrated Tolerancing Process
KC	Key Characteristic
KDE	Kernel Density Estimate
LHS	Latin Hypercube Sampling
LSC	Least-Squares Circle/Cylinder
LSL	Lower Specification Limit
MBD	Model-based Definition
MCC	Minimum Circumscribed Circle/Cylinder
MIC	Maximum Inscribed Circle/Cylinder

MZC	Minimum Zone Circle/Cylinder
NC	Numerically Controlled
NURB	Non-Uniform Rational B-Spline
(K)PCA	(Kernel) Principal Component Analysis
PCFR	Place, Clamp, Fasten, and Release
PDF	Probability Density Function
PDM	Point Distribution Model
PMI	Product and Manufacturing Information
RD(M)	Robust Design (Methodology)
SDT	Small Displacement Torsor
SMS	Skin Model Shapes
SPC	Statistical Process Control
SSA	Statistical Shape Analysis
STEP	Standard for the Exchange of Product Model Data
STL	STereoLithography File Format
TC	Technical Committee
TCA	Tooth Contact Analysis
TTRS	Technologically and Topologically Related Surfaces
USL	Upper Specification Limit
VDA	Verband der Automobilindustrie (German Assoc. of the Automotive Industry)
VDI	Verein Deutscher Ingenieure (Association of German Engineers)
WC	Worst-Case
WP	Workpiece

### *Symbols*

$a, b, m, k, K$	Cost-Function Parameters
$\mathbf{b}$	Scores, Axis Direction
$\tilde{\mathbf{b}}$	Random Scores
$c$	Cost (Tolerance-related)
$c_a$	Process Accuracy Index
$c_p$	Process Precision Index
$c_{pk}$	Process Capability Index
$c_{pm}$	“Taguchi” Index
$d$	Dimension, Distance
$d'$	Radial Distance
$d_{PS}$	Projected (signed) Distance
$d_{PSN}$	Normal Distance
$f$	Tolerance Analysis Function, Probability Density, Facet of the Convex Hull
$\mathbf{f}$	Stochastic Process with Approximation $\mathbf{f}$
$h$	Systematic Deviations (Point-wise), Height
$l_\rho$	Correlation Length
$m$	Module
$n$	(Vertex) Normal Vector

$p$	Point $\in \mathbb{R}^3$
$r$	Rotations
$t$	Tolerance, Point in Time, Translations
$w$	Assembly Direction
$x$	Point $\in \mathbb{R}^3$
$\hat{x}$	Center of a Skin Model Shape
$\hat{x}^f$	Center of a Skin Model Shape Feature
$\tilde{x}$	Point $x \in \mathbb{R}^3$ with systematic Deviations
$\bar{x}$	Point $x \in \mathbb{R}^3$ with systematic and random Deviations
$\underline{x}$	Footpoint
$C$	Total Cost (Tolerance-related), Covariance Function
$C$	Correlation Matrix
$CV$	Convex Hull Volume
$F$	Force Slope of the Assembly Force
$F_r$	Runout Error
$K$	Cost Coefficient (Quality Loss)
$L$	Quality-Loss
$P$	Precision Factor
$P$	Force Application Point of the Assembly Force
$R^2$	Coefficient of Determination
$S$	Difference Surface
$X$	Skin Model Shape as a Set of Points in $\mathbb{R}^3$
$X^f$	Skin Model Shape Feature ( $X^f \subset X$ )
$\bar{X}$	Mean (Skin Model) Shape
$\tilde{X}$	Approximation of $X$
$Y$	Key Characteristic
$\alpha$	Rotation Angle
$\alpha$	Rigid Body Transformation
$\alpha^E$	Transmission Error
$\beta$	Regression Coefficients
$\beta$	Parameter Vector
$\delta$	Moment-independent Sensitivity Measure
$\zeta$	Euclidean Distance
$\mu$	Expectation, Sample Mean
$\xi$	Linearity Coefficient
$\xi$	Gaussian Random Variables (i. i. d.)
$\rho$	Correlation Function
$\sigma$	Standard Deviation, Sample Standard Deviation
$\tau$	Small Displacement Torsor
$\chi$	Random Variables (Spatially correlated)

---

$\psi$	Standard Gaussian Random Variables (i. i. d.)
$\omega$	Weights
$\omega$	Rotations (Small Displacement Torsor) with $\alpha \equiv r_x, \beta \equiv r_y, \gamma \equiv r_z$
$\Delta p$	Displacement of $p$
$\Delta t$	Time Discretization
$\Phi$	Main Modes of Variation

*Indices*

$i, n$	1, 2, 3, ...	Counting Indices
$\psi$	1, 2, 3, ...	Assembly Steps

## Zusammenfassung

In Zeiten scharfen internationalen Wettbewerbs steigt der Druck auf Unternehmen qualitativ hochwertige Produkte mit moderaten Fertigungskosten anzubieten. Obgleich moderne Fertigungsverfahren stetig steigende Fertigungsgenauigkeiten erreichen, sind dennoch die Produktqualität und die Montierbarkeit als wesentlicher Treiber für die Fertigungskosten durch geometrische Bauteilabweichungen beeinflusst, die zwangsläufig an jedem gefertigten Bauteil zu beobachten sind. Daher existiert eine dringende Notwendigkeit für Unternehmen, diese Abweichungen und deren Auswirkungen entlang des Produktlebenszyklus zu steuern. Um dies innerhalb der Zeit- und Kostenbudgets umzusetzen, werden Produkt- und Prozessentwickler durch Toleranzsimulationsprogramme unterstützt, die die frühzeitige Vorhersage der Auswirkungen von geometrischen Bauteilabweichungen auf Produkteigenschaften ohne zeit- und kostenintensive physikalische Prototypen erlauben. Allerdings bringen bekannte Methoden und Werkzeuge zur Toleranzanalyse und deren zugrundeliegende mathematische Ansätze zur Abbildung geometrischer Abweichungen, Spezifikationen und Anforderungen schwerwiegende Nachteile in Hinblick auf die Berücksichtigung von Formabweichungen mit sich und sind nicht vollständig konform zu internationalen Tolerierungsnormen.

Als Antwort auf diese Nachteile wurde das Konzept der Skin Model Shapes als neues Paradigma für die Modellierung von Produktgeometrie unter Berücksichtigung geometrischer Abweichungen entwickelt. Es nutzt punktbasierende Modelle zur Abbildung der Produktgeometrie in Anbetracht aller Arten geometrischer Abweichungen. Die vorliegende Arbeit untersucht die Grundlagen des Konzepts der Skin Model Shapes, demonstriert seine Potentiale für die Abbildung von abweichungsbehafteter Produktgeometrie entlang des Produktlebenszyklus und zeigt wesentliche Anwendungsfelder dieses Konzepts im Kontext des Toleranzmanagements auf. Zudem wird ein Toleranzanalyseansatz auf Basis von Skin Model Shapes vorgestellt, der verschiedene Algorithmen für die Erzeugung und Verarbeitung von Bauteilrepräsentanten in diskreter Geometrie nutzt und die realistische Vorhersage der Auswirkungen von geometrischen Bauteilabweichungen auf funktions- und qualitätskritische Schließmaße erlaubt. Die vorgestellten Ergebnisse dieses Ansatzes zur Toleranzanalyse für verschiedene Problemfälle belegen, dass Formabweichungen deutlichen Einfluss auf verschiedene Produkteigenschaften haben und dass das Konzept der Skin Model Shapes sowie der vorgestellte Toleranzanalyseansatz ein theoretisch fundiertes Rahmenwerk bilden, das die Nachteile bekannter Toleranzanalyseverfahren überwindet.



## Abstract

In times of fierce international competition, the need for companies increases to deliver high-quality products manufactured at moderate costs. However, even though modern manufacturing processes offer steadily increasing accuracy, the product quality as well as the product assemblability as a main driver for the manufacturing costs are influenced by geometrical part deviations, which are inevitably observed on every manufactured workpiece. Thus, there exists a strong need for companies to manage these deviations and their effects throughout the whole product life-cycle. In order to perform this within time and cost constraints, computer-aided tolerancing tools support product and process development teams by enabling the early prediction of the effects of geometrical part deviations on product characteristics without the need for cost and time expensive physical mock-ups. However, established tools for the tolerance analysis and their underlying mathematical approaches for the representation of geometrical deviations, geometrical specifications, and geometrical requirements imply severe shortcomings regarding the consideration of form deviations and lack of a full conformance to international tolerancing standards.

As a response to these shortcomings, the concept of Skin Model Shapes has been developed as a new paradigm for the modelling of product geometry considering shape variability. It employs point-based models for the representation of part geometry considering all different kinds of geometrical deviations. The present work explores the fundamentals of the concept of Skin Model Shapes, demonstrates its potentials for the representation of product geometry considering geometrical variations along the product life-cycle, and illustrates main applications of this concept in the context of geometrical variations management. Moreover, a tolerance analysis approach utilising the concept of Skin Model Shapes is proposed, which employs various algorithms for the generation and processing of discrete geometry Skin Model Shapes and which allows the realistic prediction of the effects of geometrical variations and tolerance specifications on product key characteristics. The results obtained by this novel tolerance analysis approach for various study cases highlight, that form deviations have distinct effects on geometrical product characteristics and that the concept of Skin Model Shapes and the tolerance analysis based thereon offer a sound theoretical framework and theory, which overcomes severe shortcomings of established tolerance analysis approaches.

